Principles of Computer Science An Invigorating, Hands-On Approach

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A Logic Primer

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• Logic is the use of *deductive reasoning* to analyze an *argument*.

• Arguments are comprised of *premises* and *conclusions*.

• Premises describe the reasoning of an argument.

• A conclusion is what follows from the premises.

Truth values and connectives

- *Propositions* are statements that are either true or false.
 - E.g., "The sky is blue", "2 + 2 = 5"

• We assign *truth values*, i.e., "true" or "false", to a proposition.

• *Connectives* allow us to modify the truth value of propositions and conjoin propositions.

• Five connectives in *zeroth-order logic*: $\neg, \land, \lor, \rightarrow, \leftrightarrow$

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Logical negation

 In most instances, for any proposition 'p', it is safe to use the phrase, "It is not the case that 'p' is true", to represent the *logical negation* of 'p'.

• Problem: sentences do not have a straightforward binary conversion between non-negation and negation.

Truth table:



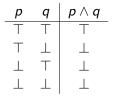
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Logical conjunction

• Represents the connection of propositions with non-symbolic words such as "and" and "but".

 Both operands of a schema must be true for the logical conjunction to be true.

• Truth table:



Logical disjunction

 Represents the truth of at least one of two schema using phrases like "or".

• Logical disjunction is inclusive-or.

• Truth table:

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Logical conditional

- Determines the truth conditions for a relationship between schema.
- When the antecedent is true and the consequent is false, the conditional is false.
- "Implication is the validity of the conditional".
- Truth table:

$$\begin{array}{c|ccc} p & q & p \rightarrow q \\ \hline T & T & T \\ \hline T & \bot & \bot \\ \bot & T & T \\ \bot & \bot & T \end{array}$$

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Logical biconditional

• True if both operands of the biconditional are the same.

• "Equivalence" is the validity of the biconditional".

Truth table:

р	q	$p \leftrightarrow q$
Т	Т	Т
Т	\perp	\perp
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Quantifiers

 In first-order logic we use quantifiers for one reason: as their name suggests, they quantify, or provide numeric amounts to, some entity.

- Universal quantifier:
 - To say that "All math majors are smart", we use *predicates* and variables: $\forall x(M(x) \rightarrow S(x))$ '
 - We say S(x) means x is smart, and M(x) represents x is a math major.

- Existential quantifier:
 - To say that "Some math majors are computer science majors", we use $\exists x(M(x) \land C(x))$ '.
 - We say C(x) means x is a computer science major.

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Identity

• Identity allows us to denote reference a particular entity.

- E.g., "Anyone who is the best computer science is Katherine Johnson":
 - $\forall x(C(x) \rightarrow \forall y((C(y) \land B(x,y)) \rightarrow x = k))'$
 - We say B(x, y) means x is better than y.

• Identity is a predicate; returns true or false if the identity relationship holds.

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Set theory

- A set S is an unordered non-duplicate collection of values.
- An element x is in S means $x \in S$.
- The number of elements in a set S is denoted as |S| also called the *cardinality*.
- A subset of S, namely S', is denoted as S' ⊆ S if all elements of S' are elements of S.
- Two sets are *equivalent* if they are subsets of each other.

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More about set theory

 The union of two sets S and T, i.e., S ∪ T, is defined as the set of elements that are in either S or T or both.

• The *intersection* of two sets *S* and *T*, i.e., *S* ∩ *T*, is defined as the set of elements that are in both *S* and *T*.

• The difference of two sets S and T, i.e., S - T, is defined as the set of elements that are in S but not in T.

 Some common mathematical sets: natural numbers N, the integers Z, the rationals Q, the reals R, and the complex numbers C.

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Functions

- Functions are maps between sets called the domain and the range.
- E.g., f(x) = x + 5 maps any number x to the set of x plus five. For instance, f(5) = 10.
- Substitute the function *parameters*, i.e., *x*, for the *arguments*, i.e., 5.
- We can write functions of multiple arguments:

$$g(x, y, z) = 3x^{2} + 4y + z$$
$$g(10, 2, 3) = 3(10)^{2} + 4(2) + 3$$
$$= 311$$

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Recursive functions

- A recursive function is a function that calls itself.
- Think of addition: if we add two values n and m, we know that n + 0 = n. To solve n + m, we should solve n + (m - 1). Then, we can propagate the result back up. Assume we know how to add and subtract one.

• E.g.,

$$add(3,4) = 3 + 4$$

= 1 + (3 + 3)
= 1 + (1 + (3 + 2))
= 1 + (1 + (1 + (3 + 1)))
= 1 + (1 + (1 + (1 + (3 + 0))))
= 1 + 1 + 1 + 1 + 3

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Data Structures

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Data structures store data!

• Different ways of storing data for performance, space optimization, and so forth.

• Many are simple, some are wildly complex.



• Arrays are contiguous blocks of storage where each block contains space for *n* elements of a given type.

• An advantage to using arrays are their quick access times.

• A disadvantage of arrays is that they are not resizable, their size must be known before creation.

• Arrays cannot store differing types; i.e., we can't store a string and an integer in the same array.

Array Lists

• Like arrays, *array lists* store elements of a type. Unlike arrays, they are resizable!

- Advantages:
 - Most implementations are quick to set up and understand, which leads to their widespread usage compared to other data structures.
 - As we said, they are resizable.
 - Insertion of new elements is easy.

- Disadvantages:
 - Easy to use, but not performant. Insertion and removal of elements is slow.

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Linked Lists

• *Linked lists* are a series of *nodes*, or elements, linked together in a chain of sorts.

- Advantages:
 - Insertion, addition, and removal is quick! No need to resize/shift values.

- Disadvantages:
 - Element/index retrieval is slow; we no longer have contiguous elements in memory.

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Stacks

- The *stack* data structure is a collection of elements that operate on the principle of last-in-first-out, or LIFO.
 - The last thing that we enter is the first thing removed.

- Advantages:
 - Fast insertion and removal operations via push and pop.

- Disadvantages:
 - Not as flexible as arrays or lists; cannot access arbitrary elements.



- The *queue* data structure is a collection of elements that operate on the principle of first-in-first-out, or FIFO.
 - The first thing that we enter is the first thing removed.

- Advantages:
 - Fast insertion and removal operations via enqueue and dequeue.

- Disadvantages:
 - Not as flexible as arrays or lists; cannot access arbitrary elements.



 Sets are similar to their mathematical counterpart; collection of unordered and non-duplicate elements.

- Advantages:
 - Easy to add and remove elements; we can also query the set for item presence.

- Disadvantages:
 - No ordering to values; no "indices" to elements of a set.

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Maps

• *Maps* are association pairs/relationships. These pairs have a *key* and a corresponding value.

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- Advantages:
 - Easy to determine whether a key exists in the map.
 - Trivial to setup a relationship between two values.

- Disadvantages:
 - No ordering to key/value pairs.



- Trees are like linked lists, but there are potentially multiple links to a node.
- Trees are recursive data structures because the elements of a tree are trees themselves.
 - E.g., binary trees are nodes with at most two children.
- Advantages:
 - Easy to describe relationships with real-world systems, e.g., mathematical structures, and even file systems.
- Disadvantages:
 - Hard to design, can become "left" or "right" leaning, decreasing performance.

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Graphs

- A graph is a tuple (V, E), where V is the set of vertices, or nodes, and E is the set of edges.
- Edges are tuples, which serve as links between vertices.
- Edges can have a direction or be bidirectional.
- Edges in a graph may also be either *weighted* or *unweighted*, denoting a "cost".
- Advantages:
 - Applicable to lots of real-world concepts.
- Disadvantages:
 - Hard to write algorithms for, and can be costly in terms of performance and space.

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Formal Languages

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• To talk about languages, we first need to define an *alphabet*.

• *Alphabets* are sets, Σ, where each element is a distinct *symbol* or a grouping of symbols.

• A *language* L over an alphabet Σ is a subset of Σ where each element is an arrangement, or a permutation, of the alphabet.

• Grammars describe the syntax of a language.

- We define a grammar G as a set of terminals T, a set of non-terminals T', and a set of production rules R.
 - A *terminal* is an atomic literal result of a production rule.
 - A *non-terminal* is a set of possible paths that a string can take in a production rule.
 - *Production rules* combine and define the relationship between terminals and non-terminals.

Backus-Naur Form grammars

• (Extended) *Backus-Naur Form* grammars are a formalism to grammatical language constructions.

• Example of an BNF grammar for a prefix notation arithmetic expression language:

```
"0" | "1" | ... | "9" | "+" | "-" | "*" | "/"
т
        ::=
               R* WS NUM OP EXPR
тי
        ...=
WS
        ..=
NUM
                ("0"
                     | "1"
                                   | "9")+
        ::=
               "+" | "=" | "*" | "/"
0P
EXPR
               "(" OP WS EXPR WS EXPR ")"
        ..=
               NUM
R*
        ::=
               EXPR
```

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Finite automata

- *Finite automata* are, in essence, very weak computers, or models of computation.
 - They describe *transitions* between *states* in some model.
 - Use input symbols belonging to an alphabet Σ .

• A deterministic finite automaton F is a quintuple $\langle Q, \Sigma, \delta, q_0, F \rangle$.

- Q is the set of states.
- δ is a transition function.
- q₀ is the start state.
- F is the set of accepting states.

Regular languages

• *Regular languages* are languages recognized by a deterministic finite automaton.

• Any DFA can be converted into a regular expression and vice versa.

• See the book for details on the syntax.

• Lexical analysis involves assigning meaning to sequences of characters.

• Example: in a string containing "1 + 23 · 41", we might *tokenize* these lexemes by assigning the token **Number** to the lexemes '1', '23', and '41'.

• We use lexical analysis primarily when designing the grammar of a programming language.

Syntactic analysis

• *Syntactic analysis*, also called *parsing*, is determining whether a sequence of tokens conform to a language grammar.

• When parsing tokens, we build data structures called parse trees, which are then converted into abstract syntax trees.

• Parse trees are hierarchical representations of tokens.

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Abstract syntax trees

- Whereas parse trees describe the syntactic structure of an input, *abstract syntax trees* explains the relationships between subtrees.
- Abstract syntax trees strip extraneous characters such as separators that do not contribute to a node in the tree.
- Example: AST of '((9 8 +) (17 81 −) ·)':



- The λ-calculus in the early 1930s is an abstract machine for modeling computation.
- We have variables, x, y, ..., z, function definitions/abstractions λv.B where v is a variable and B is a λ-calculus term, and function application (M N) where M and N are λ-calculus terms.
- Seems limited at first glance, but we can represent many computations and programs with the λ -calculus.
- Very, very, very slow from a performance standpoint, but that wasn't Alonzo Church's point!

Programming and Design

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What language for our language?

- To explore concepts in programming languages and computer science, we need to actually start programming!
- We will develop our *own* programming language in due time.
- Until then, we need to get familiar with C: the language of choice.

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- Why C?
 - It's small
 - It's fast
 - Tried and tested (to some degree)

"Hello, world!" in C

• Refer to the book for a more in-depth explanation.

```
#include <stdio.h>
```

```
int main(void) {
    printf("Hello, world!\n");
    return 0;
}
```

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Recursive functions in C

• Example of addition:

```
#include <stdio.h>
```

```
int add(int n, int m) {
  if (m == 0) {
   return n;
 } else {
   return 1 + add(n, m - 1);
 }
}
int main(void) {
 printf("%d\n", add(3, 4));
```

```
return 0;
}
```

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Conditionals

- Conditionals allow us to make decisions in our program.
- Change control flow.
- The conditional expressions must resolve to either true or false.

```
int main(void) {
    int x = 0;
    if (someCondition) {
        x = 5;
    } else if (someOtherCondition) {
        x = 10;
    } else {
        x = -1;
    }
    return 0;
}
```

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Pointers

- Passing values as arguments to functions is by value.
- Modifying that value inside the function does not change its value on the outside.

```
void swap(int x, int y) {
    int tmp = x;
    x = y;
    y = tmp;
}
```

- Pointers are locations in memory.
- We can use them to pass a reference to the variables we want to update inside the function.

```
void swap(int *x, int *y) {
    int tmp = *x;
    *x = *y;
    *y = tmp;
}
```

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Arrays

- Arrays, of course, are fixed-sized data structures.
- Size must be known at compile-time.
- Indices are indexed from zero.
- If we don't know the size at compile-time, use malloc.

```
int main(void) {
    int[] arr = new int[5];
    arr[0] = 5;
    arr[1] = 10;
    arr[2] = 20;
    arr[3] = 40;
    arr[4] = 45;
    return 0;
}
```

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Strings

- Strings are nothing more than an array of characters.
- String literals are immutable.

```
int main(void) {
    const char *s1 = "Hello, world!";
    char[] s2 = "Hello, world!";
    s2[5] = '?';
    return 0;
}
```

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Loops (1)

- While loops are for repeating a task an indeterminate number of times.
- Example: Collatz conjecture.

```
int main(void) {
  int n = ...;
  int i = 0;
  while (n != 1) {
    if (n \% 2 == 0) {
     n = n / 2;
    } else {
      n = 3 * n + 1;
    }
    i++;
  }
  return 0;
}
```

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Loops (2)

- For loops are used when we want to repeat a task a determinate number of times.
- Example: computing factorial of n.

```
int main(void) {
    int n = ...;
    int res = 1;
    for (int i = 1; i <= n; i++) {
        res *= i;
    }
    return 0;
}</pre>
```

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Structs

- Structs allow us to group data to make an "object" of sorts.
- Example: consider a student struct.

```
struct student {
  char id[256];
  double gpa;
};
int main(void) {
  struct student s1:
  strcpy(s1.id, "Katherine");
  s1.gpa = 4.0;
  struct student *s2 = malloc(sizeof(student));
  strcpy(s2->id, "Bjarne");
  s1.gpa = 3.5;
  return 0;
}
```

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Unions

• Unions let you store multiple types of values under one "umbrella".

```
union data {
  int number;
  char ch;
  char *string;
  bool val;
}
int main(void) {
  union data v1, v2, v3;
  v1.number = 5;
  v2.ch = 'A';
  v3.val = false;
  return 0;
}
```

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\mathcal{L}_{PF1} : A prefix arithmetic language

• To start things small, we will interpret a prefixed arithmetic language.

expr	::=	application
		datum
		comment
application	::=	(' expr* ')'
		'[' expr* ']'
	Í	({ expr* (})
datum	::=	number
		symbol
comment	::=	(;, ((\n')*
number	::=	('+' '-')? (digit)+ ('.' (digit) *)?
symbol	::=	symchar (symchar number)*
pf1	::=	expr+

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Representation independence with respect to ASTs

• Our programming languages will make use of Daniel Holden's *mpc* library, specifically for generating ASTs.

• Problem: what if we want to swap this library out in the future?

• **Solution:** write functions that tap into the library and use *these* functions in our interpreter.

• We will revisit representation independence multiple times.

$\mathcal{L}_{\mathsf{PF2}}$: Now with environments!

• A programming language without variables is pretty lame.

• We need to introduce the notion of *environments*.

 An environment binds identifiers to their values. E.g., (define x 5) (define y 6)

We define the association $x \mapsto 5$ and $y \mapsto 6$.

Interpretation

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\mathcal{L}_{COND} : Conditionals and Decisions

• Conditionals, as we saw in our C primer, allow us to divert program control based on decisions.

• To ease our transition, we first introduce a language with only booleans, then boolean expressions, then conditional expressions.

```
expr := application | ...
application := cond | if | ...
cond ::= '(cond' cond-clause* else-clause ')'
cond-clause ::= '[' expr ' ' expr ']'
else-clause ::= '[' else' ' ' expr ']'
if ::= '(if ' expr ' ' expr ' ' expr')'
```

$\mathcal{L}_{\text{LOCAL}}$: Local identifiers and values

• Our language is *lexically-scoped*, meaning identifiers obtain their values by when they were declared.

Introduces let, let* bindings.

(
expr	::=	application
application	::=	let $ $ letstar $ $
let	::=	<pre>'let (' let-bndg+ ')' expr</pre>
letstar	::=	<pre>'let* (' let-bndg+ ')' expr</pre>
let-bndg	::=	id''expr
_		

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\mathcal{L}_{PROC1} & \mathcal{L}_{PROC2} : Recursive procedures

 Functions, or procedures, define a callable section of code with or without parameters.

• Their definition comes through lambda, which means we can define *anonymous* and *non-anonymous* functions.

expr application proc	::=	
proc	::=	<pre>'lambda' (' id* ')' expr</pre>

$\mathcal{L}_{\text{LETREC}}$: One more time with letrec

• Sometimes, we do not want to expose a function definition into the *global namespace*.

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• **Solution:** we can define functions inside a let or let* block.

• **Problem:** these functions cannot be recursive.

• Solution: use letrec!

Different datatypes

• Restricting ourselves to working with only integers, booleans, and functions is unnecessary.

- We provide descriptions for three languages: $\mathcal{L}_{CHAR},$ $\mathcal{L}_{STRING},$ and $\mathcal{L}_{EQUAL}.$
 - \mathcal{L}_{CHAR} describes operations for working with single characters.
 - $\mathcal{L}_{\text{STRING}}$ allows us to create and manipulate strings.
 - $\bullet~\mathcal{L}_{\mathsf{EQUAL}}$ defines predicates for determining equality amongst values.

Functional Programming

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\mathcal{L}_{QUOTE} : Quoted expressions

• How can we turn code into data?

• Quoting!

• '(+ 2 3) resolves to (+ 2 3).

• What might this lead us towards?

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$\mathcal{L}_{\text{LIST}}$: Pairs and lists

- We need some type of data structure.
- Pairs contain a first and a rest.
- We create pairs using cons, and reference the elements using first and rest.
- first returns the first item of the pair.
- rest returns the second item of the pair, or the rest of the list if called on a list.

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\mathcal{L}_{QUASI} : Quasiquotes

- Quoted data is fun, but what does this evaluate to? (define x 5) (define y 6)
 - '(10 30 x 50 60 y)

'(10 30 x 50 60 y)... would it not be more sensible to resolve the x and y?

Quasiquoting and unquoting allows us to do this!
 (10 30 ,x 50 60 ,y)

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$\mathcal{L}_{VARIADIC}$: Support for variadic-argument functions

• A function that is defined to receive any number of arguments is called *variadic*.

• Under the hood, we translate these into a list of arguments.

• The function processes these arguments as if they were received a list of values.

First-class & Higher-order functions

- In our language and other functional programming languages, functions are *first-class citizens*, meaning they can be passed around as arguments to functions and returned from functions.
- Example:

$\mathcal{L}_{\text{EVAL}}$: Evaluation and application

• We have a way of converting code into data via quoting, but what about the other way around?

Two new forms: eval and apply.

• eval receives a quoted expression, or data, and attempts to evaluate it. E.g., (eval '(+ 2 3)) resolves to 5.

 apply applies a function to a list of arguments. E.g., (apply cons '(2 3)) resolves to (2 . 3).

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Accumulator-passing style

• Accumulators are values that we construct when a function is in *tail-position*.

• A function call is in tail position if it is the last action performed before a "return".

• We accumulate the result in a parameter, hence the term "accumulator-passing style".

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Continuation-passing style

- A continuation is, in effect, "the rest of a computation".
- We use continuations to direct program control to where **we** want it to go next.
- E.g., k is the continuation!

• We invoke this by (fact-cps 5 (λ (v) v))

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Nested interpreters

• Our language is now powerful enough to where we can write interpreters from within the interpreter! We call this *nested interpretation*.

- For nested languages, we need to define *recognizer* functions and *reducer* functions.
 - Recognizer functions determine whether a value represents some structure.
 - Reducer functions evaluate the structured data.

• Tons and tons of examples in the book.

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Imperative Programming

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\mathcal{L}_{SET} : Assignment statements

• C allows us to reassign variables after their initialization.

• Until now, our language does not let us.

• Doing so raises questions about the purity of our language.

expr	::=	application
application	::=	set setfirst setrest
set	::=	'set! ' symbol expr
setfirst	::=	'set-first! ' symbol expr
setrest	::=	'set-rest! ' symbol expr

$\mathcal{L}_{\mathsf{BEGIN}}$: Sequential expressions

• Assignment statements, e.g., set!, do not return a value.

• Therefore, we should add a construct that allows us to chain statements and expressions in a sequence.

• How does this help us? Closures are now easier to visualize.

begin begin expr	expr application begin	::=	application begin 'begin ' expr+
------------------	------------------------------	-----	--

\mathcal{L}_{OUT} : Fancier output

• In C we use printf for formatted output. We can output strings, booleans, integers, whatever we wish.

expr	::=	application
application	::=	printf
printf	::=	'printf' expr expr*

- Pass-by-value: pass a copy of each argument to functions.
- *Pass-by-reference:* pass a memory reference of each argument to functions. Mutating a value in the function modifies the value outside as well.
- *Lazy evaluation by name:* evaluate arguments only as they are referenced in the body of a function.
- Lazy evaluation by need: evaluate arguments only as they are referenced in the body of a function, but save the result of the expression to avoid recomputation.

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\mathcal{L}_{VECTOR} : Static data structures

• Pairs and lists are dynamic data structures; i.e., they are resizable.

• *Vectors* are like C arrays; they cannot be resized after their declaration, but provide constant lookup times.

expr	::=	application
application	::=	vector
		vector-set
	Ì	vector-get
vector	::=	'make-vector' expr
vector-set	::=	'vector-set!' id expr expr
vector-get	::=	'vector-get' id expr

\mathcal{L}_{LIB} : External libraries

• Libraries, or auxiliary files with function definitions, prevent the need to constantly rewrite functions.

 Requires careful parsing; how do we handle circular dependencies or duplicate function definitions?

::=	$application \mid \ldots$
::=	include
::=	'include ' string
	::=

\mathcal{L}_{BIGNUM} : Arbitrarily-precise numbers

• Using only 64-bit double numbers limits our program capabilities. What if we want to work with arbitrarily large values?

• No new language features aside from reworking our s-value for numbers to use *gmp* and *mpfr*.

 \bullet To simplify successive discussions, we will not use $\mathcal{L}_{\mathsf{BIGNUM}}$ following this section.

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\mathcal{L}_{IN} : Improved user input

• In C we can use getline and fgets to read strings in from different sources.

• We then parse these using sscanf or some other roughly-equivalent function.

• \mathcal{L}_{IN} adds read-string and read-number for reading strings and numbers, respectively, from standard input.

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$\mathcal{L}_{\mathsf{FILE I/O}}:$ File input and output

• Working with files is a prominent part of programming and software development.

• In C we use FILE and auxiliary functions to read data from files.

• $\mathcal{L}_{\text{FILE I/O}}$ uses the C primitives to add support for reading from and writing to files.

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\mathcal{L}_{LOOP} : An iterative approach to problem-solving

- Recursion is a great and powerful concept, but we can very easily overflow the procedure call stack.
- Moreover, some concepts are harder to understand when the only tool at our disposal is recursion.
- \mathcal{L}_{LOOP} adds a do loop construct, which functions identically to a while loop in C.

expr application do	::=	application do 'do ' expr expr
---------------------------	-----	--

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\mathcal{L}_{MACRO} : A simple macro system

- Macros are textual substitutions in code.
- We use the preprocessor in C, but we do not have such a thing in our language.
- What do macros give us? Lots of helpful language constructs that are otherwise impossible or cumbersome, e.g., *promises*.

expr	::=	application \mid	
application	::=	macro	
macro	::=	'define-macro	(' id id* ')' expr

Compilation

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An assembly primer

• We will write a small compiler for our language.

• Recall that compilers, in general, target machine-dependent assembly language; we will choose x86/64 assembly.

• Compilers are *much* faster than interpreters, hence the desire!

- Assembly is mnemonic-driven; small instructions to do small tasks. We operate primarily on *registers*: 64-bit slots for values on the CPU.
 - movq %rax, %rbx moves the data from register %rax into register %rbx.

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Compiling $\mathcal{L}_{\mathsf{PF1}}^-$ to $\mathcal{L}_{\mathsf{PF1}_{\times 64}}^-$

• Our first language supports printing only constant integer values.

expr ::= '(call (print' ', constant '))' constant ::= [0-9]+ pf1- ::= expr*

• After this we expand out to include simple binary operations and expressions.

expr ::= '(call (print' ') (constant | arithexpr) '))'
arithexpr ::= '(' binop ' ; constant ' ; constant ')'
binop ::= '+' | '-' | '*' | '/'
constant ::= [0-9]+
pf1 ::= expr*

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Compiling $\mathcal{L}_{\mathsf{PF2}}$ to $\mathcal{L}_{\mathsf{PF2}_{\mathsf{x64}}}$

• We want to support variables; let's add those! All variables are allocated on the stack. Inefficient, but simple.

expr	::=	call
		var
	Í	arithexpr
		constant
	Í	id
call	::=	<pre>(call (print' ' expr '))'</pre>
var	::=	(var ' id ' = ' expr')'
arithexpr	::=	<pre>(' binop binopval binopval ')'</pre>
binopval	::=	${call constant id};$
id	::=	[a-zA-Z]+
pf2	::=	expr*

Compiling $\mathcal{L}^-_{\mathsf{COND}}$ to $\mathcal{L}^-_{\mathsf{COND}_{x64}}$

 Before we compile conditionals, we should get boolean expressions to work.



After this we can add an if statement.

if := '(if' expr expr expr ')' cond ::= expr*		::=	(if ' expr expr expr ')'
--	--	-----	--------------------------

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Compiling $\mathcal{L}^+_{\mathsf{COND}}$ to $\mathcal{L}^+_{\mathsf{COND}_{x64}}$

• Programming languages aren't very powerful without some way to repeat an action.

• Since we do not yet have procedures, we cannot implement recursion.

-	while '(while ' expr ' ' expr ')' expr*
1	1
ł	nile ::=

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Compiling $\mathcal{L}^-_{\mathsf{PROC}}$ to $\mathcal{L}^-_{\mathsf{PROC}_{\mathsf{x64}}}$

• On the journey to functions, we will first implement *subroutines*: or functions that do not receive nor return values.

```
expr ::= proc | ...
proc ::= '(proc ' id ' ' (' id* ')' lstmt ')'
lstmt ::= expr lstmt | expr
id ::= [a-zA-Z]+
proc- ::= expr*
```

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Compiling \mathcal{L}_{PROC} to $\mathcal{L}_{PROC_{x64}}$

• Subroutines are boring!

ſ		
expr	::=	call proc
call	::=	<pre>(call ' id '(' expr* '))'</pre>
procdecl	::=	<pre>(proc ' id '(' id* ')' expr* ')'</pre>
proc	::=	expr+
procdecl	::=	(proc ' id (' id* ')' expr* ')'

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Compiling $\mathcal{L}^+_{\mathsf{PROC}}$ to $\mathcal{L}^+_{\mathsf{PROC}_{\mathsf{x64}}}$

• Some functions do not compile correctly in $\mathcal{L}_{\mathsf{PROC}_{\mathsf{x64}}}.$ We need to fix them!

• **Problem:** we delay setting argument registers until after all arguments are evaluated.

• Solution: evaluate the arguments to a function *in reverse*, push the result to the stack via pushq. Then, once all arguments have been evaluated, pop the results off the stack into the appropriate argument-registers via popq.

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Compiling \mathcal{L}_{ARRAY} to $\mathcal{L}_{ARRAY_{x64}}$

• We need a data structure to make this a truly powerful language! Let's implement stack-allocated arrays.

expr	::=	getindex setindex
decl	::=	arraydecl
arraydecl	::=	(array ' number ')'
getindex	::=	<pre>(get-index ' id ' ' expr ')'</pre>
setindex	::=	<pre>(set-index ' id ' ' expr ' ' expr ')'</pre>
array	::=	decl* expr*
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Compiling $\mathcal{L}_{\mathsf{FLOAT}}$ to $\mathcal{L}_{\mathsf{FLOAT}_{x64}}$

- We already store integers in registers; can we not do the same for floating-point values?
- Answer: **no!** Floating-point values are considerably more difficult to tackle.
- We cannot use local variables; everything is declared in the data segment.

ſ		
expr	::=	arithexpr setexpr callexpr
callexpr	::=	'(call ' id id* ')'
proc	::=	(proc main ()' (' expr+ '))'
constant	::=	number
float	::=	vardecl* proc
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Memory Management

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Stack-allocated (static) memory

• The *stack* is a small section of memory for local variable declarations (not using malloc or its derivatives).

• We also use the stack for function calls, i.e., function arguments, return values, and so forth are stored in *activation records*.

• When a function returns, its activation record is removed from the stack, thereby removing all stack-declared variables.

Heap-allocated (dynamic) memory

• The *heap* is a collection of blocks that our program can "tap into" when allocating memory at runtime.

• We have seen this with functions, e.g., malloc, calloc, realloc, strdup, and so forth.

• In C, we have to free this memory, otherwise we cause a memory leak.

Garbage collection

• Scheme is particularly tricky to allocate/deallocate memory for, because the lifetime of a function/variable is not always unknown.

• Deallocating at the wrong time will cause an undefined variable reference or crash the interpreter.

• A garbage collector keeps track of "live" heap references and deallocates these chunks when nothing points to them (i.e., they are no longer live).

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Garbage collection (cont)

• We write two garbage collectors in the book: a simple one and a reference-counted garbage collector.

- The simple garbage collector simply keeps track of the allocations made and frees them before ending the program.
 - Incredibly simple, but not very useful.

• The *reference-counted garbage collector* counts each pointer to an object in memory; once that number reaches zero for an object, it is no longer reachable and its memory is freed.

Event-Driven Programming

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Concurrent programming

- Our programs so far have been loaded in via files. What if we want to run a program and make changes on the fly?
 - We can implement a *read-evaluate-print-loop*.

• **Problem:** our system has to constantly listen for input and be ready to receive it, so how can the system also evaluate expressions in the interpreter?

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• Solution: multithreading!

Threading

• *Threads* manage separated sequence of actions for the current program to execute.

• **Problem:** multithreading opens the nasty can of worms that contains data races/race conditions. Race conditions are "competitions" for a piece of data; one thread might write into a value using an old value and another thread can then use an incorrect value.

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• Solution: mutexes and condition variables!

Multithreading and garbage collection

• Previous versions of our garbage collectors were "stop the world" garbage collectors, i.e., interpretation stops to wait for the collector to finish.

• Stop the world garbage collectors are slow!

• We can integrate multithreading into the mix and use a separate thread for our reference-counted garbage collector.