12.3.2 Terzaghi's Bearing Capacity Equation

Assuming that the bearing capacity failure occurs in general shear mode, Terzaghi (1943) expressed his first bearing capacity equation for a *strip* footing as:

$$q_{\text{ult}} = c\,N_c + \gamma_1 D_f\,N_q + 0.5\,B\gamma_2 N_\gamma \tag{12.2}$$

Here, c, γ_1, and γ_2 are the cohesion and unit weights of the soil above and below the footing level respectively. N_c, N_q, and N_γ are the bearing capacity factors that are functions of the friction angle. The ultimate bearing capacity is derived from three distinct components. The first term in Equation 12.2 reflects the contribution of cohesion to the ultimate bearing capacity, and the second term reflects the frictional contribution of the overburden pressure or surcharge. The last term reflects the frictional contribution of the self-weight of the soil below the footing level in the failure zone.

For *square* and *circular footings*, the ultimate bearing capacities are given by Equations 12.3 and 12.4 respectively.

Square: $$q_{\text{ult}} = 1.2\,c\,N_c + \gamma_1 D_f\,N_q + 0.4\,B\gamma_2 N_\gamma \tag{12.3}$$

Circle: $$q_{\text{ult}} = 1.2\,c\,N_c + \gamma_1 D_f\,N_q + 0.3\,B\gamma_2 N_\gamma \tag{12.4}$$

Remember that the bearing capacity factors in Equations 12.3 and 12.4 are those of *strip* footings. In local shear failure, the failure surface is not fully developed, and thus the friction and cohesion are not fully mobilized. For this local shear failure, Terzaghi reduced the values of friction angle and cohesion to $\tan^{-1}(0.67\,\phi)$ and $0.67\,c$ respectively.

Terzaghi neglected the shear resistance provided by the overburden soil, which was simply treated as a surcharge (see Figure 12.2). Also, he assumed in Figure 12.2 that $\alpha = \phi$. Subsequent studies by several others show that $\alpha = 45 + \phi/2$ (Vesic 1973), which makes the bearing capacity factors different from what were originally proposed by Terzaghi. With $\alpha = 45 + \phi/2$, the bearing capacity factors N_q and N_c become:

$$N_q = e^{\pi \tan\phi}\,\tan^2\left(45 + \frac{\phi}{2}\right) \tag{12.5}$$

$$N_c = (N_q - 1)\cot\phi \tag{12.6}$$

The above expression for N_c is the same as the one originally proposed by Prandtl (1921), and the one for N_q is the same as the one given by Reissner (1924). While there is a consensus about Equations 12.5 and 12.6, various expressions have been proposed for N_γ in the literature, the most used being those proposed by Meyerhof (1963) and Hansen (1970). Some of these different expressions for N_γ are presented in Table 12.2. The bearing capacity equation can be applied in terms of total or effective stresses, using c' and ϕ', or c_u and ϕ_u.