CHAPTER 1

Problem 1

a) $\frac{\partial u_1}{\partial x_1} = k \times 6x_1$, $\frac{\partial u_1}{\partial x_2} = 2kx_2$, $\frac{\partial u_1}{\partial x_3} = 0$, $\frac{\partial u_2}{\partial x_1} = 0$, $\frac{\partial u_2}{\partial x_2} = 4kx_2$

$\frac{\partial u_2}{\partial x_3} = k$, $\frac{\partial u_3}{\partial x_1} = k$, $\frac{\partial u_3}{\partial x_2} = 0$, $\frac{\partial u_3}{\partial x_3} = 8kx_3$

At the point $M(1,1,1)$ the displacement gradient matrix is:

$$10^{-3}\begin{bmatrix} 6 & 2 & 0 \\ 0 & 4 & 1 \\ 1 & 0 & 8 \end{bmatrix}$$

b) $e_{11} = 6\times10^{-3}$, $e_{22} = 4\times10^{-3}$, $e_{33} = 8\times10^{-3}$, $e_{12} = 1.0\times10^{-3}$

$e_{13} = .5\times10^{-3}$, $e_{23} = .5\times10^{-3}$

$\omega_{21} = -1.0\times10^{-3}$, $\omega_{13} = -.5\times10^{-3}$, $\omega_{32} = -.5\times10^{-3}$

c) $du_1 = [6dx_1 + 2dx_2]10^{-3}$

$du_2 = [4dx_2 + dx_3]10^{-3}$

$du_3 = [dx_1 + 8dx_3]10^{-3}$

Problem 2

a) $\frac{\partial u_1}{\partial x_1} = .006x_1x_2$, $\frac{\partial u_1}{\partial x_2} = .003x_1^2$, $\frac{\partial u_1}{\partial x_3} = 0$

$\frac{\partial u_2}{\partial x_1} = .006x_3$, $\frac{\partial u_2}{\partial x_2} = -.002x_2$, $\frac{\partial u_2}{\partial x_3} = .006x_1$

$\frac{\partial u_3}{\partial x_1} = 0$, $\frac{\partial u_3}{\partial x_2} = .012x_3$, $\frac{\partial u_3}{\partial x_3} = .012x_3 + .012x_2$

At $M(1,0,2)$ the displacement gradient matrix is:

$$\begin{bmatrix} 0 & 0.003 & 0 \\ 0.012 & 0 & 0.006 \\ 0 & 0.024 & 0.024 \end{bmatrix}$$

b) $e_{11} = 0$, $e_{12} = 0.0075$, $e_{13} = 0$, $e_{22} = 0$, $e_{23} = .015$

$e_{33} = .024$

$\omega_{21} = .0045$, $\omega_{13} = 0$, $\omega_{32} = .009$

c) $du_1 = 0.003\,dx_2$

$du_2 = 0.012\,dx_1 + 0.006\,dx_3$

$du_3 = 0.024\,dx_2 + 0.024\,dx_3$