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# Preface

The *Architectural Acoustics Handbook* attempts to summarize the present state of knowledge evolved from both the research and consulting communities in this important field. To this end, the handbook contains two Parts; Part I: *Architectural Acoustics Essentials* and Part II: *Architectural Acoustics Practice*—contributed by authorities in various subfields, though it is not always possible to establish a clean division between the two. It is meant to serve as a handy reference and a useful resource for research scientists, undergraduate and graduate students studying architectural acoustics, and for acoustic consultants and engineers who are professionally engaged in architectural acoustics practice.

As such, this volume aims to provide for audiences who are interested and engaged in frontier research with the latest progress and findings in vibrant research fields that were otherwise treated largely in specific acoustical journals. The topics and subfields covered include geometrical and wave-based room-acoustic modeling methods (Chapters 1 and 2), acoustics in long and coupled spaces (Chapters 3 and 4), measurement methods for architectural acoustics (Chapter 5), advanced room-acoustic energy decay analysis (Chapter 6), sound insulation in buildings (Chapter 7), auditory perception and auralization in rooms (Chapters 8 and 9), room-related sound representations using loudspeakers (Chapter 10) and environmental acoustics around the built environment (Chapter 11). To also serve architectural acoustics design practice, Part II of this volume provides guidance for the practical design of sound systems (Chapter 12), and heating, ventilating, and air conditioning systems in buildings (Chapter 13), as well as the acoustical design and renovations of various types of venues, including worship spaces (Chapter 14), and music performance halls, dramatic arts, and music instruction spaces (Chapter 15). To keep the book to an appropriate size, the authors were given a page limit. Most of the chapters in Part I were kept within this limit, while some chapters covering design practice in Part II were allotted more pages.

Recognizing that no single individual possesses all the expert knowledge on such a diverse field as architectural acoustics, the editor of this book wishes to extend his sincere appreciation to all the chapter authors, who alongside their professional work load, have dedicated themselves to the laborious task of presenting their respective fields of expertise in an extensive, yet compact form. We are particularly indebted to Tim Pletscher and Stephen Buda at J. Ross Publishing for their effective help and guidance in the production of this book.

Needless to say, this effort spans many years. Two esteemed chapter authors—who were delightful colleagues and highly respected acoustical consultants—passed before seeing this work published. Ronald McKay, right after delivering his chapter on *Music Performance*

*Spaces*, passed in December 2011.[1] Ewart (Red) Wetherill, who submitted his entirely completed chapter on *Acoustics in Worship Spaces* on September 1, 2013, after two rounds of thorough revisions, passed in November 2015. This book is dedicated to the memory of our esteemed colleagues, Ronald L. McKay (1932–2011) and Ewart A. Wetherill (1928–2015).

Ning Xiang, Troy, July 2016

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[1] After his passing, a number of partially completed illustrations were finished with the help of Yiqiao Hou.

# About the Editor

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# 1

# Computational Modeling of Room Acoustics I: Wave-Based Modeling

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## 1.1 ROOM ACOUSTIC MODELING

Room acoustics offer challenging problems for computational and numerical modeling. The geometry of the problem can be very large relative to wavelengths that span many orders of magnitude. At the same time, requirements for precision and accuracy might be high if the computed results are to be used for auralization or other evaluation of perceived quality.

The result is a situation where both more physically accurate wave-based methods and faster, but more approximate, geometrical methods might be necessary to cover the wide frequency range of interest with adequate accuracy. Physically motivated wave-based methods like the *boundary element method* (BEM), the *finite element method* (FEM), the *finite difference method* (FDM), and many other related variants are both computationally feasible and relevant at low frequencies or in small geometries. Analytical solutions are also available for simplified room geometries, for rooms composed of simple subdomains, and to augment partial solutions from other numerical methods.

The limited resolution of our hearing makes modeling fine details at higher frequencies less important, which also implies that these accurate but computationally costly methods might give unnecessarily precise results at high frequencies. (*Note: they are potentially precise but the input data is not available with the required precision.*) Other chapters demonstrate that different computational methods have been developed for different problems and scenarios,[1, 2] and the importance of auralization[3] and the auditory system must also be kept in mind. This chapter outlines the methods that are relevant and well-suited to modeling physical wave mechanics for low frequencies and small geometries, where effects of wave physics are important in order to get accurate results.

## 1.2 ANALYTICAL SOLUTIONS

Explicit analytical solutions to the wave equation without medium losses are available for a few geometries and types of boundary conditions, and in room acoustics where the canonical room shape is parallelepiped, as illustrated in Figure 1.1. Other potentially useful geometries often correspond to common, orthogonal coordinate systems, like the cylinder (including wedges) and the sphere (including hemispheres). These are not presented here but are available.[4] Section 2.3 also demonstrates how simplified geometries might be combined to represent more general structures. The differential equation governing linear acoustics is the second-order wave equation:

$$\nabla^2 p(\mathbf{x},t) - \frac{1}{c^2}\frac{\partial^2 p(\mathbf{x},t)}{\partial t^2} = q_S(\mathbf{x},t), \tag{1.1}$$

where $c$ is the speed of sound, and $p(\boldsymbol{x}, t)$ is the sound pressure field. The quantity $q_S(\mathbf{x},t)$ on the right-hand side is a source term, which might, e.g., be a Dirac function of space to indicate a point source. If we consider single-frequency sources, that is, sources with a time dependence of the form, $q(\mathbf{x},t) = q_S(\mathbf{x})e^{j\omega t}$, then the partial differential equation (1.1) reduces to the Helmholtz equation:

$$\nabla^2 p(\mathbf{x}) + \left(\frac{\omega}{c}\right)^2 p(\mathbf{x}) = q_S(\mathbf{x}), \tag{1.2}$$

where the function $p(\mathbf{x})$ and the source function $q_S(\mathbf{x})$ will depend on (angular) frequency $\omega$. If the geometry is one of the few canonical shapes,[5] separation of variables can be applied, and explicit solutions can be written as a classical modal summation. Furthermore, if the study is restricted to point sources (located in $\mathbf{x}_S$), then the solution can be written in the general form:

$$p(\mathbf{x}) = \frac{j\omega U_0 \rho_0 c^2}{V} \sum_{m,n,q\in[0,\infty]} \Lambda_{mnq} \frac{\Phi_{mnq}(\mathbf{x}_S)\Phi_{mnq}(\mathbf{x})}{\omega^2 - \omega_{mnq}^2}, \tag{1.3}$$

where the summation is over all combinations of integer values $m$, $n$, $q$; $\Phi(\mathbf{x})$ are the so-called mode functions which depend on geometry and boundary conditions (BC); $U_0$ is the volume velocity amplitude of the point source in $\mathbf{x}_S$; $\rho_0$ is the density of air; $V$ is the room volume;

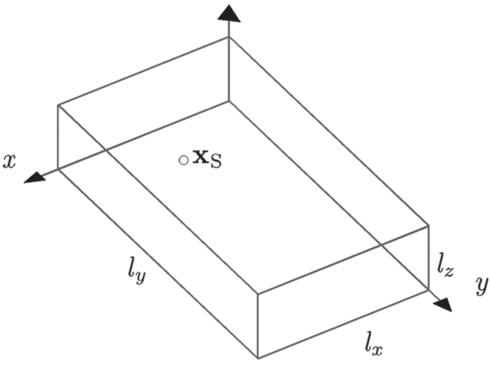

**Figure 1.1** The parallelepipedic room for which an analytical solution is available

$\omega_{mnq}$ are the so-called eigenvalues; and $\Lambda_{mnq}$ is a mode number normalization factor: 2 if two of $m$, $n$, $q$ are zero, 4 if one of $m$, $n$, $q$ is zero, and 8 if none of $m$, $n$, $q$ is zero. If $U_0 = 1$, then this solution corresponds to a transfer function in space—a Green's function—and the solution for an extended source in space can be produced by convolving the Green's function from Eq. (1.3) with the source distribution function $q_S(\mathbf{x})$.[5] The harmonic time-dependence, $e^{j\omega t}$, is left out here and in all subsequent constant-frequency expressions. It could be noted that this solution corresponds to an ideal, lossless situation, which would reach infinite amplitude if the source frequency was chosen as one of the eigenvalues $\omega_{mnq}$. The common solution to model small amounts of losses which are evenly distributed across the walls, is to introduce complex eigenvalues:

$$\underline{\omega}_{mnq} = \omega_{mnq} + \mathrm{j}\delta_{mnq}, \tag{1.4}$$

where the underline indicates a complex value, and $\delta_{mnq}$ is a loss factor, which is related to the reverberation time $T_{60}$ via $\delta = 3\ln 10 / T_{60}$.

By assuming a distributed loss, the mode functions, $\Phi(\mathbf{x})$, will be very similar to those for a lossless case. This assumption requires that losses are small, that is $\delta_{mnq} << \omega_{mnq}$, which is usually fulfilled in room acoustical cases. This small-loss assumption leads to the modal sum with losses:[6]

$$p(\mathbf{x}) = \frac{\mathrm{j}\omega U_0 \rho_0 c^2}{V} \sum_{m,n,q} \Lambda_{mnq} \frac{\Phi_{mnq}(\mathbf{x}_S)\Phi_{mnq}(\mathbf{x})}{\omega^2 - \omega_{mnq}^2 - \mathrm{j}2\delta_{mnq}\omega_{mnq}}. \tag{1.5}$$

If the eigenfunctions and eigenvalues can be computed for the geometry and boundary conditions at hand, this spectral solution can be used to compute the sound pressure amplitude for a given source frequency $\omega$.

The expression in Eq. (1.5) involves an infinite summation over three indices. At low frequencies, where the modal density is low, a single term might dominate the sum, particularly near the eigenvalues $\omega_{\mathrm{mnq}}$. Above the so-called Schröder frequency, $f_{\mathrm{Sch.}}$, however, the modal density is so high that there are large numbers of terms of similar amplitudes at any given source frequency $\omega$. The value of this important frequency is:[7]

$$f_{Sch.} = 2000\sqrt{\frac{T_{60}}{V}}, \tag{1.6}$$

where the numerical constant obviously has the unit $(\mathrm{m/s})^{3/2}$. Also below the Schröder frequency, for frequencies between eigenvalues, a large number of terms might have significant amplitudes and consequently, the sum might converge very slowly. The amplitude of higher-order terms falls off as $1/\omega_{mnq}^2$, but the number of higher-order terms is large thanks to the triple summation. A numerical example in Section 3.1 illustrates this effect. One demonstration of the slow convergence is the case when the receiver position is placed exactly at the position of the point source. This case is expected to give an infinite amplitude as a result, since the free-field (direct sound) singularity at the point source location should become imminent. But, the form in Eq. (1.5) does not seem to indicate that $\mathbf{x} = \mathbf{x}_S$ leads to any singularity. The explanation is that when $\mathbf{x} = \mathbf{x}_S$, then $\Phi_{mnq}(\mathbf{x}_S)\Phi_{mnq}(\mathbf{x})$ is always positive and consequently, the summation will diverge—for other cases, that mode function product will have alternating signs, rendering the sum convergent, albeit slowly.

Time-domain expressions, that is, impulse responses, can be found via an inverse Fourier transform of the result of Eq. (1.5), or via explicit time-domain modal summation forms.[8]

## 1.2.1 Parallelepipedic (Shoebox) Room

As previously stated, a small number of canonical shapes are *analytically* solvable by separation of variables. In practice, many rooms and buildings are essentially rectilinear in shape, so the parallelepipedic *shoebox* room is an important representative example in room acoustics. The most important boundary condition (BC) to study is the Neumann BC, formulated as:

$$\left.\frac{\partial p}{\partial n}\right|_{\text{at surface}} = 0 \Rightarrow v_n = 0, \tag{1.7}$$

which corresponds to a perfectly rigid wall with an absorption coefficient of zero. More realistic cases are discussed below, but as mentioned above, a common technique for introducing (small) losses is to maintain a lossless BC, while introducing a modal loss factor $\delta_{mnq}$. For the parallelepipedical room in Figure 1.1, with a Neumann BC on all six walls, the modal function set (the so-called eigenfunctions) has the form:[6]

$$\Phi_{mnq}(x) = \cos\frac{m\pi x}{l_x}\cos\frac{n\pi y}{l_y}\cos\frac{q\pi z}{l_z}, \tag{1.8}$$

to be used in Eqs. (1.3) and (1.5). The modal resonance frequencies (the so-called eigenfrequencies) are given by:

$$\omega_{mnq} = \pi c\sqrt{\left(\frac{m}{l_x}\right)^2 + \left(\frac{n}{l_y}\right)^2 + \left(\frac{q}{l_z}\right)^2}, \tag{1.9}$$

where $l_x, l_y$ and $l_z$ are the side lengths of the room as indicated in Figure 1.1.

## 1.2.2 Modal Solution + Propagating Waves

The 3-D eigenfunction form given in the previous section, for the case of a Neumann BC on all surfaces, might be practical to write in a form with propagating waves in one of the dimensions. As one example, we might have a locally reacting material described by an impedance $Z_{\text{wall}}$ on a single wall, e.g., at $y = l_y$, or a source distribution in the form of a vibrating wall at $y = 0$. Then the solution could be written:

$$p(x,y,z) = \sum_{m,n}\frac{\Phi_{mn}(x,z)}{\omega^2 - \omega_{mn}^2 - 2j\delta_{mn}\omega_{mn}}\left[A_{mn}e^{-jk_y y} + B_{mn}e^{jk_y y}\right], \tag{1.10}$$

$$k_y = \sqrt{k^2 - \frac{\omega_{mn}^2}{c^2}}, \quad \omega_{mn} = \frac{c}{2}\sqrt{\left(\frac{m}{l_x}\right)^2 + \left(\frac{n}{l_z}\right)^2}, \tag{1.11}$$

and the coefficients $A_q$, $B_q$ are derived to fulfill the boundary conditions at $y = l_y$ and at the source. Using this form, a number of cases can be handled, e.g., a source distribution in the wall of $y = 0$, as illustrated in Figure 1.2 can be studied, where the shaded area of the wall at $y = 0$ is vibrating as a piston. The boundary condition to fulfill at $y = 0$ is then:

$$v_z(x,0,z) = v_{z0}\sum_{m,n}V_{mn}\Phi_{mn}(x,z), \tag{1.12}$$

where the mode amplitudes $V_{mn}$ are found by expanding the source distribution function in the modal functions:

$$V_{mn} = \iint_{[0,l_x],[0,l_z]} v_z(x,0,z)\Phi_{mn}(x,z)\,dx\,dz, \tag{1.13}$$

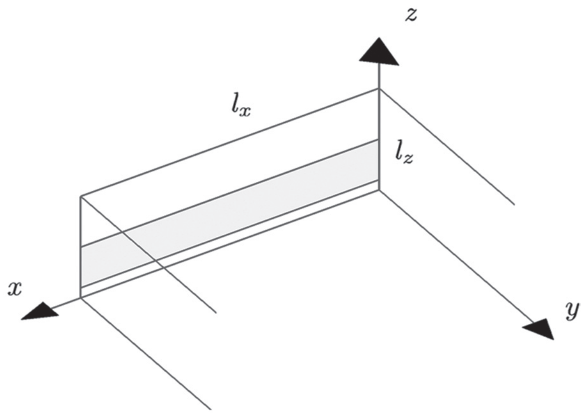

**Figure 1.2** Example with a section of a wall as a vibrating surface (shaded area)

$$\Phi_{mn}(x,z)=\cos\frac{m\pi x}{l_x}\cos\frac{n\pi z}{l_z}\,. \tag{1.14}$$

### 1.2.3 Domain Matching

Many complex geometries can be constructed of simpler, connected subdomains, for which we have individual analytical solutions. This can be as illustrated in Figure 1.3 where two parallelepipedic domains are connected.

The formulation in Subsection 2.2, with propagating waves in one of the three dimensions, can then be used to give us one description in each subdomain:

$$p^I(x,y,z)=\sum_{m,n}\frac{\Phi^I_{mn}(x,y)}{\omega^2-\left(\omega^I_{mn}\right)^2-2j\delta^I_{mn}\omega^I_{mn}}\left[A^I_{mn}e^{-jk^I_z z}+B^I_{mn}e^{jk^I_z z}\right],\quad z\in\left[0,l_z\right], \tag{1.15}$$

$$p^{II}(x,y,z)=\sum_{r,s}\frac{\Phi^{II}_{rs}(x,y)}{\omega^2-\left(\omega^{II}_{rs}\right)^2-2j\delta^{II}_{rs}\omega^{II}_{rs}}\left[A^{II}_{rs}e^{-jk^{II}_z z}+B^{II}_{rs}e^{jk^{II}_z z}\right],\quad z\in\left[l_z,l_{z2}\right]. \tag{1.16}$$

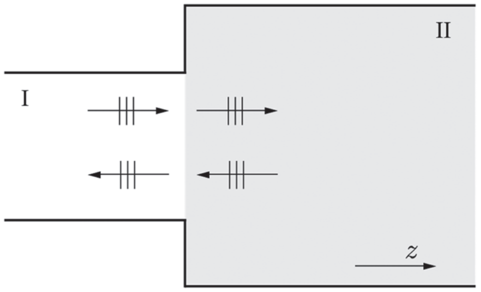

**Figure 1.3** An example geometry which can be described as one rectangular domain connected to another rectangular domain

Across the interface between the two domains, the sound pressure and its gradient must be continuous, and on parts of the interface surface, a Neumann BC might apply. The fulfillment of such matching on the interface leads to a set of equations.

To solve for the unknowns, $A_q^{I}$, $B_q^{I}$, $A_q^{II}$, $B_q^{II}$, one must decide on a maximum number of modes that will be taken into account, $n_{\max}$, $m_{\max}$, $r_{\max}$, $s_{\max}$. That choice specifies the number of unknowns and one consequently has to distribute at least that many surface sample points in which the equality should be fulfilled. Finally, a direct inversion or a solution of the overdetermined equation system via regularization must be employed.

A special case of domain matching is a decomposition of the domain under study into parallelepipedical blocks.[9] Yet another example of this approach is used in the study of ducts.[10] An analogous approach has also been used with discrete, spectral approximations of the solution in the subdomains instead of analytical solutions.[11, 12]

## 1.3 NUMERICAL SOLUTIONS

Frequently in room acoustics modeling, the geometry and boundary conditions render an analytical solution intractable, and numerical methods must be used to generate an approximate solution. Common numerical methods in room acoustics include FEMs, BEMs, and FDMs. Each can be adapted to produce solutions to the stationary problem in the frequency domain or the transient problem in the time-domain. Each method relies on discretization of the operator or solution to make the problem manageable. We present only a brief overview of each method with references to the truly massive body of literature on the subject.

### 1.3.1 Finite Difference Methods

Historically, the first methods used to generate approximate solutions to partial differential equations (PDEs) were FDMs.[13] The first applications of these methods to 3-D acoustic simulation in rooms date back to 1994.[14, 15] These early approaches have distinct origins; one grew out of methods developed for electromagnetic propagation,[14, 16] and the other comes from the approximation of wave propagation by a delay network[17] or by a transmission-line matrix.[18] All of these approaches and their variants, including the earliest ones,[13] are related, but the problem is perhaps most generally posed as a numerical solution to Eq. (1.1).

FDMs are the simplest and most accessible method described in this section, so we provide a minimal example. Typically, the problem is evaluated on a regular discrete grid in space and time, and the approximate form of the three-dimensional wave equation on the grid is given by:

$$\begin{aligned} p_{i,j,k}^{n+1} = \lambda^2 ( & p_{i+1,j,k}^{n} + p_{i-1,j,k}^{n} + p_{i,j+1,k}^{n} + p_{i,j-1,k}^{n} + p_{i,j,k+1}^{n} + p_{i,j,k-1}^{n}) \\ & + \; 2(1-3\lambda^2) p_{i,j,k}^{n} - p_{i,j,k}^{n-1}. \end{aligned} \tag{1.17}$$

Superscripts indicate temporal indices, and subscripts indicate spatial indices of grid nodes. The grid is defined by spatial and temporal steps, $\Delta x$, $\Delta t$, respectively. The time step should be chosen by fixing the spatial step so that it resolves all wavelengths of interest and setting the constant, $\lambda^2 @ c^2 \Delta t^2 / \Delta x^2 \leq 1/3$, where $c$ is the speed of sound. The Courant factor, $\lambda$, governs stability and the speed of numerical wave propagation.[13] Using the *updated Eq.* (1.17), if the pressure is known everywhere on the grid at times $n$ and $n - 1$, the pressure at time $n + 1$ may be computed from its nearest-neighbor pressure values (six for a 3-D model, four for a 2-D model). In higher-order[19, 20] or interpolated schemes,[21, 22] larger numbers of neighbor values will be involved, resulting in more accurate approximations at slightly higher computational cost. Spectral methods (Section 3.3) are in some sense a limiting case, using *all* field values to compute the derivatives.

Figure 1.4 shows how the values used in the updated equation appear on a typical grid. It is only shown in two dimensions for visual clarity. The open circle is the unknown value being computed or updated. One advantage of the FDM, illustrated in the figure, is sparsity or a locally dependent update.

Although it is relatively old, this simple update continues to be a useful tool for numerical simulation. The properties of this and its more sophisticated variants can be found, for example, in References 19 and 21–24. The practical applicability of finite-difference time-domain (FDTD) techniques is limited by high computational costs at higher frequencies such that doubling the frequency band induced 8-fold memory consumption and 16-fold computational load. In addition, the valid frequency band is limited by inherent dispersion. The actual valid band is different for each scheme, but for the basic scheme of Eq. (1.17), it gives results that are reliable up to approximately one fifth of the sampling frequency.

The previous description is derived for the scalar wave equation, whereas another popular approach solves the coupled first-order equations for pressure and particle velocity. To do so, pressure and three components of velocity are staggered in both space and time to maintain explicit time-stepping. This approach originates from the electro-magnetics literature and is often referred to as the Yee algorithm.[16, 25] However, in linear acoustics this approach is equivalent to the scalar formulation but imposes heavier computational load and memory requirements.

The overall advantage of FDMs, over others like the FEM or BEM, is realized on regular grids when the update may be applied uniformly across the domain. This also makes FDMs extremely well suited to parallelization.[26, 27]

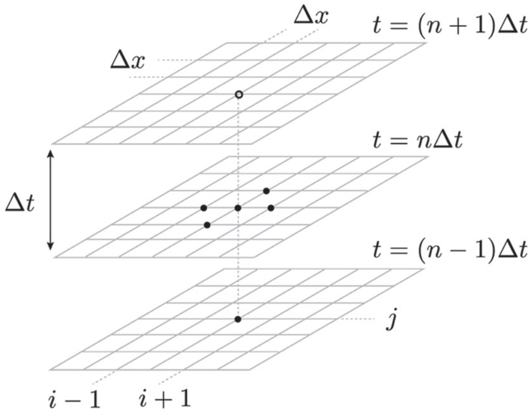

**Figure 1.4** Graphical depiction of a local, explicit, two-dimensional finite difference update for the discrete wave equation

### 1.3.2 Finite Element and Boundary Element Methods

FEMs are also based on a volume discretization of the room, but instead of discretizing the operator, the FEM uses a discrete basis set for representing the solution, often piecewise linear or piecewise polynomial. One of its distinctions from the FDM is that the mesh is often unstructured, which can increase the *geometrical* accuracy of the model. For this reason, the FEM is very popular for solving structural and vibrational problems with highly irregular geometries. It is similarly useful for acoustic problems with complex geometries, and it is most often applied to the Helmholtz equation, i.e., Eq. (1.2).[28, 29] The result in the time-harmonic case is a piecewise linear or piecewise polynomial approximation to the eigenfunctions of Eq. (1.3); however, time-domain FEM solutions are also possible.[30]

BEMs typically approximate solutions to the Helmholtz-Kirchoff integral, which involves the pressure gradient on the boundary and the Green's function. Discretizing boundary surfaces leads to matrices of Green's functions that relate a source to boundary elements. The matrix only scales with surface area instead of volume, so it may be smaller than a finite difference or a finite element matrix for the same problem. However, the matrix is dense in contrast with the sparse matrices of the FEM and FDM, so even with fewer elements, it may be more expensive. One approach that is applicable in some cases is to use fast multipole methods, which can exploit the regularity of dense BEM matrices.[31–33]

### 1.3.3 Spectral Methods

Spectral methods, whether associated with discretized solutions or operators, are characterized by exponential convergence rates.[24] FDMs and FEMs converge at polynomial rates, but methods using suitable spectral differentiation and spectral elements can converge much faster. As with higher-order or interpolated difference methods, greater accuracy allows coarser discretization, which then leads to less computation. The trade-off essentially reduces to smaller, but denser matrix operators.

The advantage of spectral methods is achieved by expanding the solution or operator into an orthogonal basis, such as a trigonometric series or the Chebyshev polynomials, and the approximation is done in this spectral domain. Using Fourier or Chebyshev bases, spectral methods have been adapted to irregular geometries through coordinate transformations and domain matching,[9, 11, 12] analogous to Section 2.3. Especially when problems are limited by memory storage, spectral methods are potentially a good alternative to finite difference or finite element methods. However, the problems for which they are applicable typically coincide with domains where an analytical solution is available.

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